The Pisa Mathematics Assessment—An Insider's View

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INTRODUCTION

In this paper I will describe what it is that the Programme for International Student Assessment (PISA) mathematics assessment aims to measure, and I will present a case regarding the potential benefits that could flow from a pedagogy that takes the PISA mathematics challenges seriously. Other potential uses of PISA data will also be briefly canvassed.

The Value of Comparative Studies

While national assessment programs such as the National Assessment of Educational Progress (NAEP) provide data with which to track changes in educational outcomes at the school and district levels, and which can generate state-level and inter-state comparisons as an external context within which those local changes may be interpreted, studies such as PISA present an opportunity to obtain strong international comparative measures. How do the proficiencies of 15-year-old students in the United States stack up next to those of 15-year-olds in comparable countries? How do they compare with the proficiencies of students in countries that are economic competitors of the United States? The PISA mathematics assessment provides comparative measures of student proficiencies based on a set of objectives that are widely seen to be relevant to students nearing the end of the compulsory school years. A key question therefore is: *are these proficiencies and objectives valued here*?

Objectives of PISA Mathematics

Those objectives are based firmly on a set of mathematical competencies that encapsulate the kinds of skills needed by mathematically literate individuals in a modern society:

- the ability to recognise the existence of mathematical elements within a situation presenting a challenge;

- the ability to pare the situation down to its elements, enabling a separation of those mathematical features from the context, and to define a mathematical problem, the resolution of which might help to answer the challenge;
- the ability to call on relevant mathematical knowledge and to apply that knowledge confidently and correctly to forge a mathematical solution to the problem at hand;
- the ability to relate that solution back to the original situation, thereby ensuring that the solution makes sense and genuinely addresses the challenge—that is, the ability to recognise the extent and limitations of the solution;
- the ability to communicate the outcomes to others; and
- the ability to step outside the process and exercise control mechanisms that help direct thought and action to achieve the desired outcome.

Further, PISA presents a model for a set of teaching and learning objectives that could form an important part of mathematical instruction at least in the junior and middle years of secondary school, and possibly more widely than that. If school mathematics programs took these objectives to heart, arguably students would be better equipped to make confident and effective use of the mathematical skills they have learned at school as they negotiate the challenges of their life as citizens. Fortunately, these objectives are consistent with the vision of mathematics education articulated through the United States' Principles and Standards for School Mathematics from the National Council of Teachers of Mathematics (NCTM).

The Challenge

How much teaching and learning time is devoted to presenting students with problems set in authentic* contexts? To what extent does the U.S. education system value the investment of effort by individuals and groups of students in grappling with such problems, pulling a problem apart, hunting for the knowledge that would help to make the problem tractable, imposing an analytical structure, applying mathematics to the problem, developing solutions, evaluating the solutions, communicating the solutions to others, considering ways of implementing the solutions, and talking about that whole exploration and discovery process?

If that investment of time is not made, is it reasonable to expect students to do well at assessments that call on those skills and processes? Could we expect students to go into postsecondary courses with the confidence and competence to interact productively with problems that demand those skills? Could employers reasonably expect their young workers to have those skills?

THE DETAILS—WHAT IS "PISA MATHEMATICS"?

The most recent publication of the PISA mathematics framework (Organisation for Economic Co-operation and Development [OECD], 2006) was in the context of the PISA 2006 survey administration. That framework, in common with the frameworks for the other PISA assessment domains, defines the field of interest, and uses three main organising ideas: contexts, content, and processes.

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* PISA mathematics uses "authentic" to refer to problems for which there is genuine interest in the solution and for which the purpose of using mathematics is to solve the problem, in contrast to contrived applications that are presented mainly for the purpose of practising particular skills. The most authentic problem contexts are those for which the context itself influences the solution and its interpretation, and which therefore require students to consciously and actively connect the problem context with the solution.

First, it is important to keep in mind that PISA is an assessment program, not an instructional program. Its primary aim is to provide the basis for generation of strong, reliable, and useful international comparative measures of the extent to which 15-year-old students can make productive use of their accumulated knowledge to solve certain kinds of problems. Note also that we are not talking about 10-year-olds or about students at the end of their senior high school years, but about an assessment program for students at the end of their compulsory schooling years who in a legal sense may be ready to leave school and take their place in society as young adult citizens.

The general PISA rhetoric emphasises phrases like “preparedness for life,” “competencies for tomorrow’s world,” a focus on “skills that are essential for full participation in society,” and on “young people’s ability to use their knowledge and skills to meet real-life challenges” (for example, see OECD, 2007, pp. 16–21). Those responsible for the development of the mathematics framework and its associated assessment instruments have interpreted and implemented this rhetoric through several fundamental features: the use of contextualised problems, the particular treatment of mathematical content, and the central place of mathematical competencies.

Contextualised Problems

PISA mathematics is largely about solving contextualised problems; about applying one’s knowledge in context. Operationalising the rhetoric about “skills for life” involves establishing a definition of the domain to be assessed, and developing test items that are relevant to the perspective of the 15-year-old. Test developers look for situations in which typical 15-year-olds will come across mathematics, or the opportunity to use mathematics, as they negotiate their daily routines. They recognise that in fact mathematics does not always jump out and shout “here

I am." More often, students need to do some thinking or work before they might even realise that their mathematical knowledge could be used to meet a challenge that is presented.

An example of such a problem is presented in

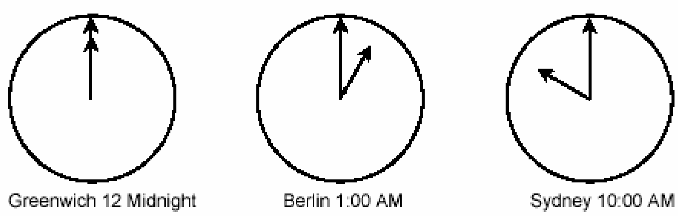
Figure 1. This item is from the PISA 2003 assessment administration, and it along with all other publicly released PISA items can be found in OECD (2009). Its solution requires students to transform information about personal schedules into a mathematical form, and to perform time-zone calculations, which PISA would regard as lying firmly within the mathematics sphere. Real-world constraints have to be considered and practical decisions made.

Figure 1. Is This Obviously a Mathematics Item?

INTERNET RELAY CHAT

Mark (from Sydney, Australia) and Hans (from Berlin, Germany) often communicate with each other using "chat" on the Internet. They have to log on to the Internet at the same time to be able to chat.

To find a suitable time to chat, Mark looked up a chart of world times and found the following:



Mark and Hans are not able to chat between 9:00 AM and 4:30 PM their local time, as they have to go to school. Also, from 11:00 PM till 7:00 AM their local time they won't be able to chat because they will be sleeping.

When would be a good time for Mark and Hans to chat? Write the local times in the table.

Place	Time
Sydney	
Berlin	

The mathematics framework recognises several different contexts:

Personal contexts: These are the situations and challenges that arise in a student's direct day-to-day personal experiences and from his or her perceptions of those experiences. The *Internet Relay Chat* item presented in

Figure 1 is set in a *personal* context.

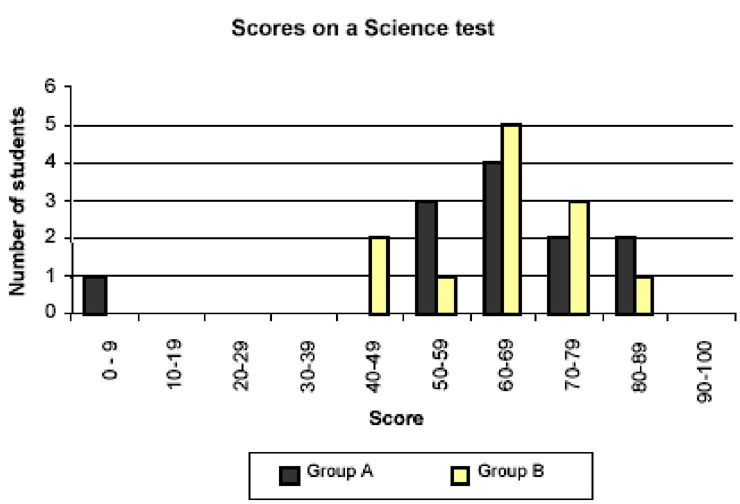
Educational/occupational contexts: Slightly removed from the immediately personal contexts and perceptions are those experiences arising from the way individuals spend their "working" day—usually at school or in employment. Some PISA problems are set in occupational contexts or in contexts that one might only come across in school. The item *Science Tests* presented in Figure 2 is set in an *educational* context.

Figure 2. A PISA Item Set in an *Educational* Context

SCIENCE TESTS

The diagram below shows the results on a Science test for two groups, labelled as Group A and Group B.

The mean score for Group A is 62.0 and the mean for Group B is 64.5. Students pass this test when their score is 50 or above.



Looking at the diagram, the teacher claims that Group B did better than Group A in this test.

The students in Group A don't agree with their teacher. They try to convince the teacher that Group B may not necessarily have done better.

Give one mathematical argument, using the graph, that the students in Group A could use.

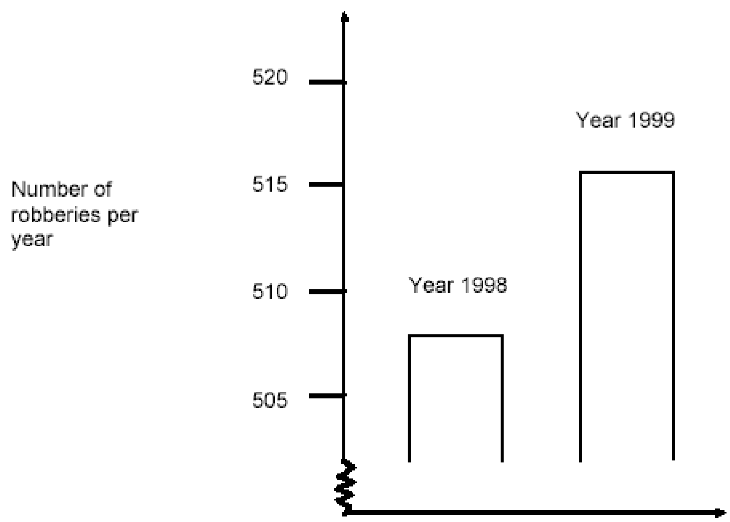
Public contexts: A little further removed from individuals' personal and internal life, and the experiences they meet through their daily work program, are the experiences one comes across by interacting with the community and the external world. Advertising and the media provide many rich opportunities to apply mathematical knowledge to evaluate claims and counter-claims from public discussions. The item *Robberies*, shown in Figure 3, is set in such a context.

Figure 3. A PISA Item Set in a *Public* Context

ROBBERIES

A TV reporter showed this graph and said:

"The graph shows that there is a huge increase in the number of robberies from 1998 to 1999."



Do you consider the reporter's statement to be a reasonable interpretation of the graph? Give an explanation to support your answer.

Scientific contexts: The final context type used in PISA for mathematics problems includes items framed in a purely scientific way. For many citizens problem contexts of this type would sometimes, but not always, seem rather removed from their immediate personal experience; for example, items set in a medical (scientific) context may seem far removed to some people but may often have direct personal relevance. Two PISA items set in a *scientific* context, *Space Flight* and *Heartbeat*, are presented in Figure 4.

Figure 4. Two PISA Items Set in *Scientific* Contexts

SPACE FLIGHT

Space station Mir remained in orbit for 15 years and circled Earth some 86 500 times during its time in space.

The longest stay of one cosmonaut in the Mir was around 680 days.

Approximately how many times did this cosmonaut fly around Earth?

A 110
B 1 100
C 11 000
D 110 000

HEARTBEAT

For health reasons people should limit their efforts, for instance during sports, in order not to exceed a certain heartbeat frequency.

For years the relationship between a person's recommended maximum heart rate and the person's age was described by the following formula:

Recommended maximum heart rate = 220 - *age*

Recent research showed that this formula should be modified slightly. The new formula is as follows:

Recommended maximum heart rate = 208 – (0.7 × *age*)

A newspaper article stated: "A result of using the new formula instead of the old one is that the recommended maximum number of heartbeats per minute for young people decreases slightly and for old people it increases slightly."

From which age onwards does the recommended maximum heart rate increase as a result of the introduction of the new formula? Show your work.

In school, students know when they are in a mathematics class. They know that most of the lesson will be taken up with learning mathematics, practising their mathematical skills, solving mathematical problems, and so on. They will be expecting to use numbers, to work with shapes, to look at relationships among variables, to draw graphs and charts, and to work with data. They will typically be studying a particular topic, and will be accumulating a set of skills and tools that they are told will be relevant to that topic.

Outside of the classroom, however, stuff just happens. A person will need to read and interpret a bus timetable, or will notice a shadow and think about the way it moves, or will want to make a shopping decision about what size container provides best value for money, or will need to measure some materials needed to make something, or to estimate how long it might take to get from *point A* to *point B.* Individuals will interpret the signals with which they are bombarded in a variety of ways, depending on their knowledge and previous experience, their state of mind, their level of confidence, and many other factors that will come into play. The individual will need to impose some structure on the signals to decide if he or she will respond, and how to respond. *Is this a mathematical problem? Can I interpret these signals in such a way that mathematics might be helpful?* These may not always be the first questions that come to mind, but sooner or later, a mathematically literate citizen will ask a question like these, implicitly or explicitly, as a precursor to activating his or her mathematical knowledge in seeking to solve the problem.

PISA mathematics recognises first and foremost that solving problems demands active participation of the problem solver: he or she must recognise the existence of a problem, must make decisions about how it might be solved, must recognise what mathematical knowledge may be relevant, and may frequently have to manipulate the problem to make it amenable to the application of that knowledge. There are many other skills needed by an effective problem solver, which I will mention further a little later.

The Treatment of Mathematical Content

The importance of the contexts in which individuals meet problems is also reflected in the way that PISA mathematics treats mathematical content. This treatment of mathematical content is another feature that distinguishes PISA as an assessment program from an approach that might be expected in an instructional context where a finer level of detail is needed. PISA treats mathematics content as fitting into four broad phenomenological categories that reflect the contexts in which problems might arise.

Quantity is used to include the mathematical knowledge that might be brought to bear on problems that involve counting, calculating, measuring, estimating, and generally working with quantifiable variables. Questions like “how much …?” or “how big …?” or “how many …?” or “how fast …?” will generally be of this type because they give rise to the application of skills involving estimating, counting, measuring, calculating, and the like. The item *Space Flight* in Figure 4 is a *Quantity* item, as is *Rock Concert*, presented later in Figure 5.

Space and shape is used to describe the many problem situations that involve manipulation of physical objects, or shapes; an appreciation of patterns observed in physical objects; and understanding of the relationships between objects and their physical representations. Questions that require interpreting a map, diagram, or photograph fit here, as do questions involving thinking about, manipulating, or analysing two- or three-dimensional objects. Knowledge of geometry will be particularly relevant.

Change and relationships is the area covering mathematical functions and relations: dealing with relationships among variables and understanding change processes such as growth and decay. Situations such as tides, seasonal changes, climatic changes, plant growth, and investment growth are just a few contexts that can give rise to challenges demanding an understanding of functional relationships, and application of mathematical skills of graphing and interpreting graphs, manipulating algebraic expressions of functional relationships, and so on. The item *Internet Relay Chat* in

Figure 1 and the item *Heartbeat* in Figure 4 are examples of items involving the *Change and relationships* area.

Uncertainty deals with problem situations that involve working with data, interpreting probabilistic information, and imagining what might happen on the basis of what has already happened. Statistical knowledge and skills will be relevant here, as will understanding of probability. The *Robberies* item in Figure 3 is set in the *Uncertainty* area, as is *Test Scores* in Figure 2.

PISA does not usually ask students to *"solve the following equation…,"* nevertheless, to solve a problem from the real world that is about the relationships among variables, students will often need to formulate, manipulate, and solve an equation. The *Heartbeat* item in Figure 4 presents this kind of demand. It is interesting to note that this item was found to be one of the most difficult items when first administered in the PISA 2003 field trial survey instrument.

Similarly, PISA will generally not ask students to *"carry out the following calculation…"* but the solution of many PISA problems will require the successful and accurate implementation of standard calculation processes. The *Space Flight* item in Figure 4 presents this kind of demand.

PISA presupposes that students possess a certain toolbox of mathematical knowledge, skills, and techniques, which they will have accumulated over several years of schooling. However, PISA does not attempt to directly assess student mastery of particular mathematical techniques, or of the topics typically studied in the mathematics classroom. Instead, it presents students with some stimulating problem in context, together with a challenge to identify and marshal whatever mathematical knowledge they have that is relevant to solving the problem. The question is, can they work out which knowledge is useful in a given situation, and can they make effective use of that knowledge?

Mathematical Competencies—the Lynchpin of PISA Mathematics

When confronted with a problem, what does the problem-solver need to do? A student may be able to factorise cubic polynomials, perform complex integration, and multiply four-digit numbers in his or her head. But we know that this will not necessarily be enough. A mathematically literate person needs to be able to activate certain mathematical competencies to make the required connections between a problem as presented and the mathematical knowledge required to solve it.

The PISA mathematics framework uses a set of competencies based on a formulation by mathematicians and mathematics educators working in the Danish KOM project (for a summary, see Niss, 2003). The set used by PISA comprises these: *thinking and reasoning mathematically; problem posing and solving; argumentation; mathematical modelling; representing mathematical entities; communicating mathematics; using symbolic, formal, and technical language and operations;* and *using mathematical aids and tools*. This set has much in common with many national curriculum statements, such as the NCTM standards and the United Kingdom's National Curriculum.

These competencies are directly related to the set of process skills referred to in the introduction to this article, which is a description of a process known as *mathematical modelling*. If you ask a university-level teacher of mathematics what it is that he or she most wants to instil in his or her students, mathematical modelling skills will usually come up early in the discussion. Mathematical modelling is fundamental to the way in which applied mathematicians work, and in its most serious form is the stuff of industry and science these days.

But not only is modelling central to the work of mathematicians and scientists, it is easy to see the direct relevance of the steps listed earlier to the thinking and action required by nonmathematicians when they are confronted with a problem for which mathematics may be helpful. The PISA mathematics framework seeks to emphasise the importance of the components of mathematical modelling in a way that is appropriate for 15-year-old students.

One must first recognise that a problem exists. Sometimes a problem is clearly stated, but not always. In one's day-to-day life, a potentially challenging stimulus may at first appear simply as an interesting observation. There may need to be some *interpretation* of whatever stimulus is presented to recognise the existence of some problem state. To begin to tackle the problem mathematically, the solver needs to *think analytically* about the situation, to identify different aspects of the problem as presented to work out which are the essential elements, and to distinguish these from aspects that are not central. The nonessential elements can be discarded or set aside. The solver then needs to express the essence of the problem in mathematical terms, the key step of establishing a *mathematical model*. This might involve invoking some specifically *mathematical representation* of the problem, such as a table, graph, or equation. It could involve applying *mathematical symbols* to express the problem. It will always involve some form of *mathematical reasoning.* Once the problem has been expressed in a mathematical form, the solver can then search his or her toolbox of *mathematical techniques* to find the most relevant one, apply it to the problem, and *find a mathematical solution*. This might include identifying suitable *mathematical aids and tools* that will help, such as using a measuring instrument, a spreadsheet or other computer program, or a calculator. The solution found should be tested, involving some *mathematical argumentation* or *proof.*

But that is by no means the end of the process. The solver must then check the solution found against the original problem, and decide whether the solution genuinely answers the original question. Is the solution realistic? Can it be applied to solve the original problem? Is it a complete solution? Does it satisfy those “nonessential” elements of the original problem situation that were set aside? Are some aspects of the problem not yet resolved? Is some further generalisation of the solution possible? Often the act of *communicating* the solution found will help the solver to evaluate the solution, and this may lead to the realisation that a further refinement of the *mathematical formulation* of the problem is needed, leading back to the beginning of the solution process. The mathematical modelling process is frequently iterative, involving several cycles of *interpretation*, *formulation*, *solution*, *evaluation*, *reformulation*, *solution refinement*, and so on.

Of course in a time-limited assessment like PISA, it is not possible to present problems that represent the entire modelling cycle as described above. What typically happens is that test items are devised that demand the application of some part or parts of the whole process. Ideally, when looking across the whole set of items in a PISA mathematics assessment, one would look to see all or most of the modelling cycle represented. To do so remains an ongoing challenge to PISA item developers.

Figure 5. A PISA Item Involving Modelling

ROCK CONCERT

For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing.

Which one of the following is likely to be the best estimate of the total number of people attending the concert?

A 2 000
B 5 000
C 20 000
D 50 000
E 100 000

A number of released items demonstrate this point. *Rock Concert*, a mathematics item from PISA 2003 shown in Figure 5, presents a context that would be familiar to many 15-year-olds, and provides the opportunity to devise a model for the amount of space that a person might occupy while standing. With the multiple choice format used in this item, this could be done by postulating an area for each person, multiplying it by the number of people given in each of the options provided, and comparing the result with the conditions given in the question. Alternatively, the reverse could be done, starting with the area provided and working backward using each of the response options, to the corresponding space per person, and deciding which one best fits the criteria established in the question. The student must think clearly about the relationship between the model he or she uses and the resulting solution on the one hand, and the real context on the other, to validate the model used and to be sure he or she has chosen the correct answer.

Another item that aims to assess just part of the modelling cycle is presented in Figure 6.

Figure 6. Assessing Part of the Modelling Cycle

> **HEARTBEAT 2**
>
> The formula *recommended maximum heart rate* = 208 – (0.7 × *age*) is also used to determine when physical training is most effective. Research has shown that physical training is most effective when the heartbeat is at 80% of the recommended maximum heart rate.
>
> Write down a formula for calculating the heart rate for most effective physical training, expressed in terms of age.

This is a second question associated with the item presented earlier (in Figure 4). In this example, students are presented with a mathematical expression that describes the situation discussed in the item and are asked to manipulate the model (that is, to apply some algebra skills, as well as some mathematical analysis and thinking skills) to produce an expression that describes a slightly changed context.

The *Internet Relay Chat* item presented earlier in

Figure 1 also provides the opportunity for students to engage with an important part of the modelling cycle, namely the reflection phase. In this case the strictly mathematical work involved is a set of time-zone calculations. However, the heart of the problem is in identifying and evaluating various real-world constraints, and using these consciously to evaluate and refine the mathematical solution.

In looking across these three examples, we see the demand for students to formulate a model, to manipulate a model, and to engage seriously in reflection and analytical thinking about the links between the context and the model. And this is done in a way that is accessible to 15-year-olds, with respect to both the contexts used and the level of mathematics required.

Additional Background Information—Student Attitudes

In addition to the mathematical knowledge and competencies possessed by students, attitudes toward mathematics form another important set of factors relevant to the proficiencies that students bring to bear in solving mathematical problems. It is stating the obvious that the attitudes held by individuals have a powerful effect on their behaviour. This is no truer anywhere than in the area of mathematics, where we see in the community a high prevalence of negative attitudes toward mathematics. Many people are less than confident in their mathematical ability. Mathematics anxiety and even mathematics phobia are not uncommon, and we believe that such negative attitudes inhibit the activation of mathematical knowledge and skills. On the other hand, confidence in one's mathematical knowledge and ability can significantly enhance a person's willingness to engage with mathematics.

While not part of the PISA mathematics definition of mathematical literacy, it is well recognised that "attitudes and emotions such as self-confidence, curiosity, feelings of interest and relevance, and the desire to do or understand things" are important contributors to the likelihood with which an individual will put his or her literacy skills into practice (OECD, 2006, p. 73).

For this reason, the PISA survey includes background questionnaires that gather relevant information about students (and about their schools), which is collected to help increase understanding of the proficiency measures obtained, and to inform the development of intervention strategies. In the PISA 2003 survey, in which mathematics was the major assessment domain, data captured from the background questionnaires were used to generate indices of *interest and enjoyment of mathematics, self-concept in mathematics, self-efficacy in mathematics, anxiety in mathematics,* and about various learning strategies used or preferred by students.

IMPLICATIONS FOR THE MATHEMATICS CLASSROOM

The PISA mathematics framework and assessment carry with them some implied directions for teaching and learning in the mathematics classroom.

Contextualised Problems and Mathematical Modelling

The first is the value placed on contextualised problems. It seems self-evident that students who have had the opportunity to grapple with problems set in authentic contexts will learn to handle them better as a result of practice. Moreover, experience in grappling with problems set in authentic contexts might reasonably be expected to contribute to the kind of learning that would set the scene for later life, such that one's mathematical knowledge and skills might be more easily activated as an adult interacting with the challenges and opportunities he or she meets. However, it may not always be so easy to take this to the classroom, for a number of reasons. First, the curriculum constraints in different places vary quite a lot in the extent to which they already accommodate applications of mathematics as a key aspect of mathematics teaching and learning. Most curriculum authorities seek to place very strong emphasis in their mathematics curriculum on achieving full coverage of a large number of formal mathematics topics, and on practising the skills and techniques learned. The highest priority is frequently given to the perceived needs of students headed for further study in mathematics and other sciences, and the idea of spending significant time on problems set in authentic contexts can tend to conflict with that priority.

More extended mathematical investigations can also provide opportunities for the kind of learning that would reasonably be expected to improve general problem-solving skills. However, this requires significant teacher expertise and a lot of preparation.

Taking this one step further would be an increased emphasis on mathematical modelling. Even though this is highly valued by scientists and mathematicians, it is difficult to do well with secondary school students, it takes a lot of teacher preparation to do well, and it is easy for such activities to be squeezed out in favour of more traditional teaching and learning activities that may appear to have a more direct and immediate link to course objectives.

Nevertheless, if we accept that the skills that contribute to mathematical modelling are important for everyone, then we need to think hard about how to provide opportunities for students to practise parts of the modelling cycle, to apply that knowledge to extended modelling activities, and to explore mathematics in authentic contexts. Moreover, part of giving students the opportunity to explore these important aspects of mathematics will be the way in which discussion occurs in the classroom. These mathematical processes need to be made explicit, and one way to do this is through discussion. The teacher will have a critical role in asking the right questions to help students develop and articulate their thoughts about how they interpret situations, what mathematics might be relevant, how the mathematics might be used to solve the problem, and how to evaluate the solutions they produce. Discussions of that kind will lead to the students being better able to understand, manage, and control their problem-solving efforts.

Emphasising the Mathematical Competencies

A second implication lies in the way in which mathematical competencies are valued and promoted in mathematics classes. The PISA mathematics assessment has exposed useful information about what competencies 15-year-olds can and cannot activate.

For example, test items that require students to activate significant *communication* competencies challenge many 15-year-olds. Both the receptive and expressive aspects of communication need to be considered here. Teachers are already familiar with the fact that *word problems* are usually more difficult than the corresponding *naked number problems*. The more a problem calls for interpretation and analytical thinking for the student to identify and apply the appropriate mathematical knowledge, the more difficult the problem will be. And at the other end of the solution process, problems requiring students to write down and explain their solution are also found to be more difficult than those that simply require a numeric answer. Why do 15-year-old students find this so difficult? Do we teach our students to question and explore? Do we give them opportunities to expose their thinking process by explaining their solutions? Do we press them to think about and explain the extent to which their solution is complete? We should not expect students to automatically have those skills unless they are taught and practised in the classroom.

How comfortable and confident are students with connecting the *mathematical language* that is appropriate for expressing a problem and the language of the problem context? To what extent do students understand the link between the language of the problem and the symbols and other formal mathematical language used to express it mathematically? To what extent do we present different *mathematical representations* of problems, and teach students to understand the relationships among those representations, showing how certain representations may be better for exposing particular aspects of the problem? This can involve something as simple as translating a word problem into a graph, or a mathematical formula, or both, and consciously comparing the features of each representation—different features of the problem will likely be more easily seen in one or other of these representations. The ability to move comfortably between different representations of a problem can provide a powerful means of analysing it.

To what extent do teaching practices emphasise the importance of *mathematical reasoning, argumentation,* and *proof*? These are central among those competencies that set mathematics apart from other knowledge domains. Do we value these competencies in our mathematics classes? Do our students have an appreciation of the power of mathematical reasoning and proof? PISA test developers have not found it easy to write test items that demand these skills yet are accessible to 15-year-old students; nevertheless, this is seen as an area deserving priority in the test development process.

The *Science Tests* item shown in Figure 4, along with four other released PISA items, are discussed further in an article in the *Mathematics Teacher* (McCrone et al., 2008) that may well have had widespread circulation within the U.S. mathematics education community. In that article, the results of U.S. students are compared with those of students from Canada and students from Mexico on several PISA mathematics items. The discussion there highlights the way particular items draw on the PISA mathematical competencies, and the data shown provide some evidence of the proficiency of U.S. 15-year-olds relative to the OECD average, in the activation of these competencies. One message from that analysis is that there is room for improvement among U.S. students.

The important lesson coming from this discussion is that rather than focusing only on what particular technical mathematical knowledge students are acquiring through their mathematics classes, attention should also be given to the ways in which students are able to apply that knowledge, to the mathematical process competencies they are able to bring to bear on the solving of problems set in authentic contexts.

POSTSCRIPT

As well as thinking about the goals and underpinnings of the PISA mathematics assessment, and the implications this may have for teaching and learning in the mathematics classroom, there are certainly broader policy questions that the very rich PISA dataset can help to answer. Some of these can be broached right away, and some might be answered more fully only if the United States were to consider expanding its PISA sample somewhat. I present a number of such questions.

How do different subgroups in the U.S. student population fare in relation to one another? To what extent do observed proficiency differences relate to measures of socioeconomic status? To what extent do they relate to the incidence of public versus private schooling? How does the U.S. experience with factors such as these compare with the patterns observed in other countries that may be of interest?

One of the lessons from international studies that has been drawn to my attention is that at grade 4 U.S. students are doing very well comparatively; at grade 8 they are not quite so strong but still doing OK; but once they get to high school the story is not so positive. Is that so? What factors might be contributing to that drift, or if this is not seen as a factual assessment, why would this perception be held by intelligent observers? What policy adjustments are needed to improve the situation? How do the state-level figures on grade-based comparisons in the United States compare with figures in other comparable countries, especially in federal countries like Germany, Canada, and Australia?

Finally, a point made earlier in this presentation: states have their own testing programs and NAEP is available to be used as a check on whether gains made on state tests over time are real or merely artifacts. But is it appropriate to use PISA data as an international benchmark? Can PISA legitimately be seen as providing a check on whether gains claimed at the national level in the United States are real, and whether they are substantial compared with the changes that are happening in other countries? For me the answer is yes, provided due attention is paid to the important caveat that PISA does not aim to provide measures specific and limited to national curriculum goals. Rather, it does provide strong measures of student capabilities at the end of the compulsory schooling years that are in large part a reflection of the knowledge and skills students have accumulated over several years of participation in a national education system.

PISA results therefore cannot be used as direct curriculum benchmarks in any participating country: PISA assesses aspects of student literacy that are not completely covered by all (or indeed by any) national curricula. So it is the responsibility of national expert panels and curriculum authorities to examine which of the dimensions are and which are not covered in their schools, then to decide on whether the ones not covered should not be taught or should be given greater prominence. International studies are a powerful instrument to help raise important questions about national curricula. When a country discovers that its students are unable to do things that students in other countries can do, the crucial question is: do our students need these things too, to be able to thrive in our modern society? If the answer is yes, then it would probably be wise to have a serious look at our curriculum—to improve it in case these things are covered but not learned, or to include them if they are not covered.

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